

# Atomic quantum dots coupled to BEC reservoirs

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We study the dynamics of an atomic quantum dot, i.e. a single atom in a tight optical trap which is coupled to a superfluid reservoir via laser transitions. Quantum interference between the collisional interactions and the laser induced coupling to the phase fluctuations of the condensate results in a tunable coupling of the dot to a dissipative phonon bath, allowing an essentially complete decoupling from the environment. Quantum dots embedded in a 1D Luttinger liquid of cold bosonic atoms realize a spin-Boson model with ohmic coupling, which exhibits a dissipative phase transition and allows to directly measure atomic Luttinger parameters.

A focused laser beam superimposed to a trap holding an atomic Bose-Einstein condensate (BEC) [1] allows the formation of an *atomic quantum dot* (AQD), i.e., a single atom in a tight trap [2, 3] which is coupled to a reservoir of Bose-condensed atoms via laser transitions. This configuration can be created, e.g., by spin-dependent optical potentials [4], where atoms in the dot and the reservoir correspond to different internal atomic states connected by Raman transitions. Atoms loaded in the AQD will repel each other due to collisional interactions. In the limit of strong repulsion, a collisional blockade regime can be realized where either one or no atom occupies the dot, while higher occupations are excluded. Below we will study the dynamics of such an AQD coupled to a BEC reservoir: as the key feature we will identify the competition between two types of interactions, namely the coupling of the atom in the dot to the BEC density fluctuations via collisions, and the laser induced coupling to the fluctuating condensate phase. Depending on the choice of interaction parameters, they can interfere destructively or constructively, providing a tunable coupling of the dot to the phonons in the condensate in the form of a spin-Boson model [5]. In particular, an essentially complete decoupling of the dot from the dissipative environment can be achieved, realizing a perfectly coherent two-level system. This interference and tunability of the coupling of the dot to the environment occurs for condensates in any dimensions. A particularly interesting case is provided by a 1D superfluid reservoir [6], i.e., a bosonic Luttinger liquid of cold atoms [7], where the system maps to a spin-Boson model with ohmic coupling. The tunable dot-phonon coupling then allows the crossing of a dissipative quantum phase transition [5, 8], and can serve also as a novel spectroscopic tool to measure directly atomic Luttinger parameters.

Let us consider cold bosonic atoms with two (hyper-fine) ground states  $a$  and  $b$  (Fig. 1). Atoms in state  $a$  form a reservoir of atoms in a superfluid phase, held in a shallow trapping potential  $V_a(\mathbf{x})$ . The AQD is formed by trapping atoms in state  $b$  in a tightly confining potential  $V_b(\mathbf{x})$  produced, e.g., by a focused laser beam induced potential or by a deep optical lattice potential which is

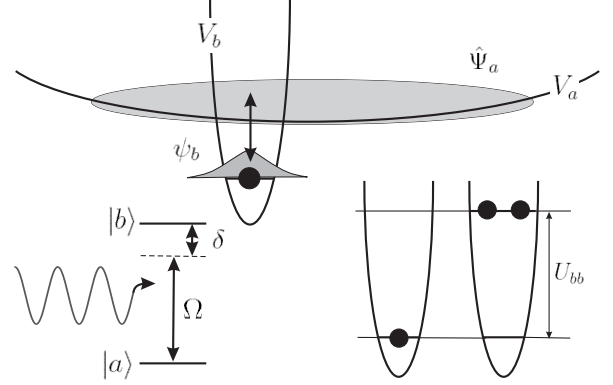


FIG. 1: Schematic setup of an atomic quantum dot coupled to a superfluid atomic reservoir. The Bose-liquid of atoms in state  $a$  is confined in a shallow trap  $V_a(\mathbf{x})$ . The atom in state  $b$  is localized in tightly confining potential  $V_b(\mathbf{x})$ . Atoms in state  $a$  and  $b$  are coupled via a Raman transition with effective Rabi frequency  $\Omega$ . A large onsite interaction  $U_{bb} > 0$  allows only a single atom in the dot.

only seen by atoms in state  $b$  [4]. Within the standard pseudopotential description, the collisional interaction of atoms in the two internal levels  $\alpha, \beta = a, b$  is described by a set of coupling parameters  $g_{\alpha\beta} = 4\pi a_{\alpha\beta} \hbar^2 / m$  with scattering lengths  $a_{\alpha\beta}$  [9] and atomic mass  $m$ . We assume that the reservoir atoms are coupled via a Raman transition to the lowest vibrational state in the AQD, where spontaneous emission is suppressed by a large detuning from the excited electronic states. Thus, following arguments analogous to those in the derivation of the Bose-Hubbard model of cold atoms in an optical lattice [4], we obtain an effective Hamiltonian,

$$H_b + H_{ab} = \left( -\hbar\delta_0 + g_{ab} \int d\mathbf{x} |\psi_b(\mathbf{x})|^2 \hat{\rho}_a(\mathbf{x}) \right) \hat{b}^\dagger \hat{b} + \frac{U_{bb}}{2} \hat{b}^\dagger \hat{b}^\dagger \hat{b} \hat{b} + \int d\mathbf{x} \hbar\Omega (\hat{\Psi}_a(\mathbf{x}) \psi_b(\mathbf{x}) \hat{b}^\dagger + \text{h.c.}), \quad (1)$$

Here  $\hat{\Psi}_a(\mathbf{x})$  is the annihilation operator for an atom  $a$  at the point  $\mathbf{x}$ , while  $\hat{\rho}_a(\mathbf{x}) = \hat{\Psi}_a^\dagger(\mathbf{x}) \hat{\Psi}_a(\mathbf{x})$  is the associated operator for the density. The operator  $\hat{b}$  destroys a  $b$ -atom in the dot in the lowest vibrational state with

wave function  $\psi_b(\mathbf{x})$ . The first term in Eq. (1) includes the Raman detuning  $\delta_0$ , and the collisional interactions between the  $b$ -atoms with the reservoir. The second term describes the onsite repulsion  $U_{bb} \sim g_{bb}/l_b^3 > 0$  between atoms in the dot with  $l_b$  the size of the ground state wave function. The last term in (1) is the laser induced coupling between  $a$  and  $b$  atoms with effective Rabi frequency  $\Omega$ . In writing (1) we exclude coupling to higher vibrational states in the dot, assuming that these states are offresonant [4].

At sufficiently low temperatures the reservoir atoms in  $a$  form a superfluid Bose liquid with an equilibrium liquid density  $\rho_a$ . The only available excitations at low energies are then phonons with linear dispersion  $\omega = v_s|\mathbf{q}|$  and sound velocity  $v_s$ . With the assumption that the number of condensate atoms inside the dot is much larger than one,  $n_a = \rho_a l_b^3 \gg 1$ , i.e.  $l_b$  is much larger than the average interparticle spacing in the BEC reservoir, the quantum dot is coupled to a coherent matter wave and the Bose-field operator can be split into magnitude and phase,  $\hat{\Psi}_a(\mathbf{x}) \sim \hat{\rho}_a(\mathbf{x})^{1/2} e^{-i\hat{\phi}(\mathbf{x})}$ . The dimensionless proportionality coefficient is the bare condensate fraction, which depends on non-universal short distance properties of the Bose-liquid. This representation does not require the existence of a true condensate and can be also used to describe both 2D and 1D superfluid systems [11]. The density operator can be expressed in terms of the density fluctuation operator  $\hat{\Pi}$ :  $\hat{\rho}_a(\mathbf{x}) = \rho_a + \hat{\Pi}(\mathbf{x})$ , which is canonically conjugate to the superfluid phase  $\hat{\phi}$ . In the long wavelength approximation the dynamics of the superfluid is described by a (quantum) hydrodynamic Hamiltonian [11]

$$H_a = \frac{1}{2} \int d\mathbf{x} \left( \frac{\hbar^2}{m} \rho_s |\nabla \hat{\phi}|^2(\mathbf{x}) + \frac{m v_s^2}{\rho_a} \hat{\Pi}^2(\mathbf{x}) \right), \quad (2)$$

where  $\rho_s$  is the density of the superfluid fraction (at zero temperature  $\rho_s = \rho_a$ ). The quadratic Hamiltonian (2) is easily diagonalized by introducing standard phonon operators  $b_{\mathbf{q}}$  via the following transformation:

$$\hat{\phi}(\mathbf{x}) = i \sum_{\mathbf{q}} \left| \frac{m v_s}{2 \hbar \mathbf{q} V \rho_a} \right|^{1/2} e^{i\mathbf{q} \cdot \mathbf{x}} (b_{\mathbf{q}} - b_{-\mathbf{q}}^\dagger) \quad (3)$$

$$\hat{\Pi}(\mathbf{x}) = \sum_{\mathbf{q}} \left| \frac{\hbar \rho_a \mathbf{q}}{2 v_s V m} \right|^{1/2} e^{i\mathbf{q} \cdot \mathbf{x}} (b_{\mathbf{q}} + b_{-\mathbf{q}}^\dagger) \quad (4)$$

with  $V$  the sample volume. Accordingly, the Hamiltonian (2) takes the form of a collection of harmonic sound modes:  $H_a = \hbar v_s \sum_{\mathbf{q}} |\mathbf{q}| b_{\mathbf{q}}^\dagger b_{\mathbf{q}}$ . Since the excitations of a weakly interacting Bose-liquid are phonon-like only for wavelengths larger than the healing length  $\xi \geq l_b$ , the summation over the phonon modes is cutoff at a frequency  $\omega_c = v_s/\xi \approx g_{aa}\rho_a/\hbar$ .

In the following we consider the collisional blockade limit of large onsite interaction  $U_{bb}$ , where only states

with occupation  $n_b = 0$  and 1 in the dot participate in the dynamics, while higher occupations are suppressed by the large collisional shift. This situation and its description is analogous to the Mott insulator limit in optical lattices [4, 10, 12]. The requirements are that the Raman detuning and Rabi couplings are much smaller than  $U_{bb}/\hbar$ , which provides a small parameter to eliminate the states with higher occupation numbers perturbatively. As discussed below, a Feshbach resonance can help in achieving this large  $U_{bb}$  limit [13]. Thus, the quantum state of a dot is described by a pseudo-spin-1/2, with the spin-up or spin-down state corresponding to occupation by a single or no atom in the dot. Using standard Pauli matrix notation, the dot occupation operator  $\hat{b}^\dagger \hat{b}$  is then replaced by  $(1 + \sigma_z)/2$  while  $\hat{b}^\dagger \rightarrow \sigma_+$ . Furthermore, the dominant coupling between the AQD and the superfluid reservoir arises from the long wavelength phonons. For wavevectors  $|\mathbf{q}|/l_b \ll 1$ , the phonon field operators may be replaced by their values at  $\mathbf{x} = \mathbf{0}$ . Neglecting the density fluctuations in the Raman coupling (see below) and an irrelevant constant, the Hamiltonian (1) is simplified to

$$H_b + H_{ab} = \left( -\frac{\hbar \delta}{2} + \frac{g_{ab}}{2} \hat{\Pi}(\mathbf{0}) \right) \sigma_z + \frac{\hbar \Delta}{2} (\sigma_+ e^{-i\hat{\phi}(\mathbf{0})} + \text{h.c.}). \quad (5)$$

Here  $\Delta \sim \Omega n_a^{1/2}$  is an effective Rabi frequency with  $n_a^{1/2}$  the bosonic enhancement factor due to the BEC reservoir, and with a proportionality coefficient which depends on the bare condensate fraction and the explicit form of the wavefunction  $\psi_b$ . The form of the Rabi coupling in (5) only applies for  $\Delta \ll \omega_c$ , which is the interesting regime in the spin-Boson model discussed below. In (5) the detuning has been renormalized to include a mean field shift and a shift due to the virtual admixture of the double occupied state in the dot,  $-\hbar \delta_0 + g_{ab} \rho_a + (\hbar \Delta)^2 / (-2U_{bb}) \equiv -\hbar \delta$ .

Moreover, the validity of the above model requires a strong collisional interaction of atoms in the AQD,  $g_{bb} \gg g_{aa}$ . This follows from the inequalities  $n_a \gg 1$ ,  $\Delta \ll \omega_c$  and the single occupancy condition  $\hbar \Delta \ll U_{bb}$ . A magnetic or (Raman laser induced) optical Feshbach resonance in the  $b$  channel will assist in achieving this limit [13]. These resonances arise from coupling to a bound molecular state in an energetically closed collisional channel. In the case of an optical Feshbach resonance, for example, the onsite interaction due to the laser induced Raman coupling of two  $b$  atoms in the dot to a molecular bound state leads in Eq. (1) to the replacement  $U_{bb} \rightarrow U_{bb}^{(\text{res})} = U_{bb} + g^2/\delta_m$ , where the second term describes the resonant enhancement with  $g$  an effective Raman Rabi frequency and  $\delta_m$  the detuning from the molecular resonance is (valid for  $g < |\delta_m|$ ). A finite lifetime of the molecular state, e.g. due to collisions with  $a$  atoms, introduces a width  $\delta_m \rightarrow \delta_m - i\Gamma/2$  [21]. Thus for detunings  $|\delta_m| \gg \Gamma$  we have  $U_{bb} \rightarrow U_{bb}^{(\text{res})} + i\gamma_m/2$

with  $\gamma_m = (g^2/\delta_m^2)\Gamma \ll U_{bb}^{(\text{res})}$ . Returning to the Hamiltonian (5) we see that besides the resonantly enhanced onsite interaction we have a non-Hermitian loss term  $\sim (\hbar\Delta/U_{bb}^{(\text{res})})^2\gamma_m$  which is strongly suppressed in the collisional blockade limit.

Eventually after a unitary transformation  $H = S^{-1}(H_a + H_b + H_{ab})S$  with  $S = \exp(-\sigma_z i\phi(\mathbf{0})/2)$  the dynamics of the AQD coupled to the phonons of the superfluid reservoir is described by a spin-Boson type Hamiltonian [5]

$$H = -\frac{\hbar\Delta}{2}\sigma_x + \sum_{\mathbf{q}} \hbar\omega_{\mathbf{q}} b_{\mathbf{q}}^\dagger b_{\mathbf{q}} + \left( -\delta + \sum_{\mathbf{q}} \lambda_{\mathbf{q}}(b_{\mathbf{q}} + b_{\mathbf{q}}^\dagger) \right) \frac{\hbar\sigma_z}{2} \quad (6)$$

The coherence in the reservoir is crucial for the most important feature of our cold atom version of the spin-Boson model (6), namely the fact that the collisional interactions and those arising from the coupling of the Rabi term to the condensate phase add coherently in the amplitudes of the total phonon coupling

$$\lambda_{\mathbf{q}} = \left| \frac{m\hbar\mathbf{q}v_s^3}{2V\rho_a} \right|^{1/2} \left( \frac{g_{ab}\rho_a}{mv_s^2} - 1 \right). \quad (7)$$

In particular, we see that for a repulsive inter-species interaction,  $g_{ab} > 0$ , both contributions interfere destructively: the effect of the phonon excitation in a laser-driven transition  $a \leftrightarrow b$  can be precisely cancelled by the change in the direct “elastic” interaction between the liquid and the  $b$  atoms, as described by the first term in Eq. (5). In a weakly interacting gas  $mv_s^2 = \rho_a g_{aa}$  and thus the coupling constants  $\lambda_{\mathbf{q}}$  vanish at  $g_{ab} = g_{aa}$ . At this special point, we have thus formed the analogue of a “charge” qubit of solid state physics [14], i.e., a coherent superposition of occupation and non-occupation of the dot, which - at vanishing detuning - exhibits perfect Rabi oscillations of the AQD’s occupancy. Notice that the simple form (7) is valid only for  $|\mathbf{q}|l_b \ll 1$ , while for larger values of the momentum the interaction starts to decrease proportional to the Fourier transform of the wavefunction  $\psi_b$ .

We discuss now the importance of the neglected density fluctuations in the Rabi term and their effect at the decoupling point. After the unitary transformation above, these fluctuations give rise to a perturbation  $H' = \frac{\hbar\Delta}{2} \frac{\hat{\Pi}(\mathbf{0})}{2\rho_a} \sigma_x$  which is small compared to the collisional interaction in Eq.(5) provided that  $\hbar\Delta \ll \rho_a g_{ab}$ . Now unless  $g_{ab}$  is much smaller than  $g_{aa}$ , this condition is precisely equivalent to the condition  $\Delta \ll \omega_c$  discussed above. In particular, if we are at the decoupling point, the only remaining interaction with the phonon bath is given by  $H'$ . For  $\delta = 0$  the Hamiltonian can then be rewritten as

$$H = -\frac{\hbar\Delta}{2} \left( 1 - \frac{\Pi(\mathbf{0})}{2\rho_a} \right) \sigma_x. \quad (8)$$

This is an independent Boson model which can be diagonalized exactly [15]. Since the phonons now no longer couple to the  $b$  atom occupation  $\sigma_z$  and, moreover, the coupling constants are proportional to the small parameter  $\Delta/\omega_c$ , one obtains perfect Rabi oscillations except for a small reduction in amplitude by a factor  $\exp(-\gamma(\Delta/\omega_c)^2) \approx 1$ , where  $\gamma \lesssim (\rho_a a_{aa}^3)^{1/2} \ll 1$  is proportional to the small gas parameter.

Let us now turn to discuss the properties of system when it can be described by the Hamiltonian Eq. (6). The system is characterized by the effective density of states

$$J(\omega) = \sum_{\mathbf{q}} \lambda_{\mathbf{q}}^2 \delta(\omega - \omega_{\mathbf{q}}) = 2\alpha\omega^s, \quad (9)$$

where  $\alpha \sim (g_{ab}\rho_a/mv_s^s - 1)^2$  is the dissipation strength due to the spin-phonon coupling and  $D = s$  the dimension of the superfluid reservoir. In the standard terminology [5],  $s = 1$  and  $s > 1$  correspond to the ohmic and superohmic cases, respectively. In the superohmic case, the resulting dynamics of the AQD is a damped oscillation at vanishing detuning  $\delta = 0$ , consistent with the result of simple Bloch equation analysis. It may be observed by following the population of the atoms  $b$  in the presence of the laser coupling. The associated frequency  $\hat{\Delta}$  and damping  $\Gamma$  can also be obtained by measuring a weak field absorption spectrum, which would exhibit the oscillations with frequency  $\hat{\Delta}$  as a line splitting, and  $\Gamma$  as the linewidth.

A much richer dynamics appears for ohmic dissipation (Fig. 2). In this case, the system exhibits a zero temperature dissipative phase transition, as a function of the dissipation strength, at a critical value  $\alpha_c = 1$  for  $\Delta \ll \omega_c$  [5]. In the symmetry broken regime  $\alpha > \alpha_c$  the occupation probability of the  $b$  atom will exhibit a finite jump from  $(1 - m_s)/2$  to  $(1 + m_s)/2$  as the detuning  $\delta$  is changed across zero. The spontaneous polarization  $m_s$  is a function of  $\alpha$  approaching  $m_s \simeq 0.9$  for  $\alpha \rightarrow \alpha_c^+$  [18] and  $m_s = 1$  for  $\alpha \gg \alpha_c$ . As a result the  $b$  occupation probability is almost unity for any  $\alpha > \alpha_c$ . Instead in the regime  $\alpha < \alpha_c$ , at vanishing detuning, the average population of the  $b$  atoms is  $1/2$ . In particular, for  $\alpha < 1/2$  one has damped Rabi oscillations. In terms of the characteristic frequency scale  $\Delta_r(\alpha) = \Delta(\Delta/\omega_c)^{\alpha/(1-\alpha)}$ , the effective Rabi oscillation frequency  $\hat{\Delta}$  and damping rate  $\Gamma$  are given by  $\hat{\Delta} = \cos \eta \cdot \Delta_r(\alpha)$  and  $\Gamma = \sin \eta \cdot \Delta_r(\alpha)$  with  $\eta = \pi\alpha/(2(1 - \alpha))$  [5]. This result holds as long as  $T \lesssim T_* = \hbar\Delta_r/\alpha$ . At higher temperatures the dynamics is incoherent and no oscillations should be visible. For  $1/2 < \alpha < 1$  the Rabi oscillations disappear and the behaviour is completely incoherent [16]. Only numerical results are available for the dynamics in this regime [17].

The ohmic spin-Boson model is achieved by embedding the AQD in an atomic quantum wire as realized recently in [19]. For transverse harmonic trapping both the temperature  $T$  and the chemical potential need to be

smaller than the frequency of the transverse confinement  $\omega_{\perp}$ . The proper description of a 1D superfluid in terms of the hydrodynamics Hamiltonian (2) is based on the Haldane-Luttinger approach [7]. The definitions of the phase and the density fluctuations in terms of the phonon operators are given by Eqs.(3,4) with  $v_s = \hbar\pi\bar{\rho}_a/(mK)$ , where  $\bar{\rho}_a \sim \rho_a l_{\perp}^2$  is the 1D density of the cloud and  $l_{\perp} = (\hbar/m\omega_{\perp})^{-1/2}$  the transverse ground state size. The Luttinger parameter  $K$  depends on the density of the liquid through the combination  $\gamma_a = m\bar{g}_{aa}/\hbar^2\bar{\rho}_a$ . The corresponding 1D interaction constant  $\bar{g}_{aa}$  is related to the 3D scattering length  $a_{aa}$  [20] and  $\bar{g}_{aa} = 2\hbar\omega_{\perp}a_{aa}$  as long as  $a_{aa} \ll l_{\perp}$ .

Using Eq.(7) for the coupling constants  $\lambda_q$ , we find

$$\alpha = \frac{1}{8K(\gamma_{aa})} \left( \frac{\gamma_{ab}K(\gamma_{aa})^2}{\pi^2} - 1 \right)^2 \quad (10)$$

where  $\gamma_{ab} = m\bar{g}_{ab}/\hbar^2\bar{\rho}_a$ . Similar to the discussion above, the interference between the direct interaction and the Rabi-transition terms in the Hamiltonian (5) leads to the disappearance of AQD-phonon coupling at  $\gamma_{ab} = \pi^2/K^2$ . The Luttinger liquid parameter  $K$  can thus be determined by tuning the AQD to the decoupling point via a change in the known interaction constant  $\gamma_{ab}$ . Thus, observation of the dynamics of the dot coupled to a Luttinger liquid provides a novel tool to measure  $K$  directly. For a weakly interacting 1D liquid  $\gamma_{aa} \ll 1$  the Bogoliubov approximation applies, giving  $K = \pi/\sqrt{\gamma_{aa}}$ . In recent experiments [19] values  $\gamma_{aa} \geq 1$  have been reached in an array of independent one-dimensional tubes. In this case the critical value  $\alpha_c = 1$  for the dissipative phase transition is reached at  $a_{ab}/a_{aa} \sim 1$ .

In conclusion, we have shown that an AQD coupled to a superfluid reservoir leads to spin-Boson model with tunable parameters. The present model may be readily extended to arrays of AQDs, where the pseudo-spins can interact with “host” liquid in a collective manner.

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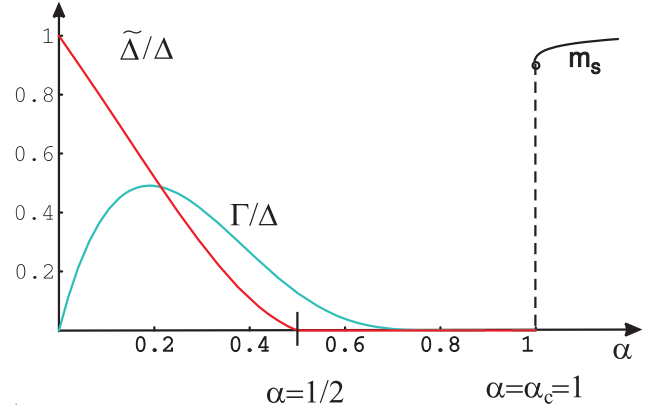


FIG. 2: The oscillation frequency  $\tilde{\Delta}$  and the damping rate  $\Gamma$  as functions of the coupling strength  $\alpha$ . For the damping in the range  $1/2 < \alpha < 1$  we used the approximate expression  $\Gamma = (1/2 - \alpha) \tan(\pi\alpha)\Delta_r(\alpha)$  [22]. The dissipative phase transition shows up as a jump of size  $m_s$  in the occupation as the detuning  $\delta$  is changed from small negative to positive values.

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